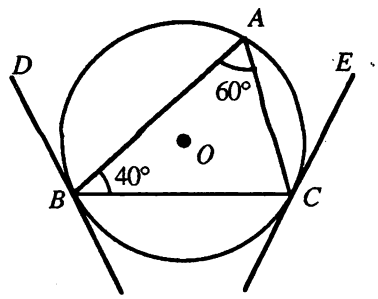
SAN DIEGO MATH LEAGUE **MIDDLE SCHOOL DIVISION, ROUND 2-A**
January 2010

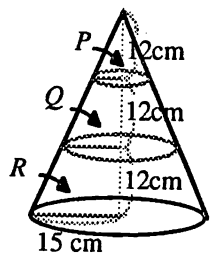
1. Given $\sqrt{3.2} = 1.789$ (namely we take 1.789 as an approximation of $\sqrt{3.2}$) what is value of $\sqrt{320}$?

2. When a fair 6-sided die is thrown twice and the two numbers shown are multiplied, what is the probability that the product is 6?

3. The figure shows the measure (in terms of degrees) of two angles of ΔABC and two lines $\overline{BD}$ and $\overline{CE}$ are tangent to the circumscribed circle points at B and C. What is measure (in degree) of angle $\angle BCE$?

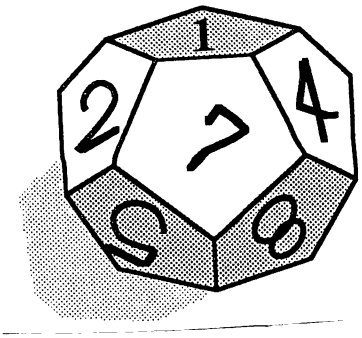


4. A cone with base radius of 15 cm and a height of 36 cm is divided into three sections P, Q and R, by planes passing through it parallel to the base, each 12 higher starting from the base. Denote by $\text{Vol}(P), \text{Vol}(Q), \text{Vol}(R)$ the volumes of P, Q and R. What is $\text{Vol}(P) : \text{Vol}(Q) : \text{Vol}(R)$?



5. Find all possible values of $(x-2)^2 + (x+3)^2$ if $(x+3)(x-2) = 0$.

6. The numbers 1 to 12 are inscribed on the faces of a die having the shape of a regular geometric solid with 12 faces (a regular dodecahedron). Determine the probability that the number on the top, when we roll it, is either a multiple of 2 or a multiple of 3.



7. Consider two circles O and O' of different sizes. If they are externally tangent the distance between the centers is 5 cm. If they are internally tangent the distance between the centers is 1 cm. What are the radii of these circles (in cm)? (Your answer should a pair of numbers. The ordering is not necessary.)

8. Two fair six-sided dice are rolled. Let a be the number on the first die and b be the number on the second die. Combine these two numbers into the coordinates of a point (a, b) and plot (a, b) on the coordinate plane. Determine the probability that the point lies inside the circle with its center at the origin and radius 4.

9. The perfect squares $1, 4, 9, 16, 25, \cdots$ are joined into a string of digits $1491625\cdots$. What is the 50-th digit in the string?

10. Point P is outside of circle O. Line segments $\overline{PA}$ and $\overline{PB}$ are tangent to circle O at points A and B, respectively. If angle $\angle P$ measures 60 degrees, what fraction of the circumference of circle O is the minor arc AB? Express your answer as a common fraction.

11. Let $2x + 3y = 72$. How many ordered pairs (x, y), with x, y being whole numbers (non-negative integers), satisfy the equation?

12. How many of the divisors of 8! are larger than 7!?

13. Three consecutive positive odd integers x, y, z satisfy $y^2 - x^2 = 344$ and $z^2 - y^2 > 0$. What is the value of $z^2 - y^2$?

14. If $3!5!x! = 10!$, find x.

15. Alex buys 20 shares of stock at \$20 per share. The shares increase in value by 20%. He then sells 10 shares. After the shares decrease in value by 10%, he buys another 20 shares. The shares then all increase by 50%. What is the value of Alex's 30 shares now?

16. For how many positive integers less than 100 is the product of the number's distinct prime factors equal to 6?

17. Rectangle $ABDE$ is inscribed in a circle. The length of segments $\overline{AB}$ and $\overline{AE}$ are 48 cm and 20 cm respectively. Point C is on the arc (of the circle) between B and D with $BC = CD$. What is the perimeter of the pentagon $ABCDE$ in centimeters? (Expressed your answer in simplified radical form.)

18. What is the ratio of the area of a square inscribed in a semicircle with radius r to the area of a square inscribed in a circle of radius r? Express your answer as a common fraction.

19. If $\frac{x}{y} = \frac{3}{4}$, $\frac{y}{z} = \frac{2}{3}$ and $\frac{z}{w} = \frac{5}{8}$, find $\frac{x+y+z}{w}$.

20. Triangle ABC has a right angle at B, side lengths $AB = 1$ and $BC = 2$. The bisector of $\angle BAC$ meets BC at D. What is BD (namely the length of the segment $\overline{BD}$)? Express your answer in simplified radical form.